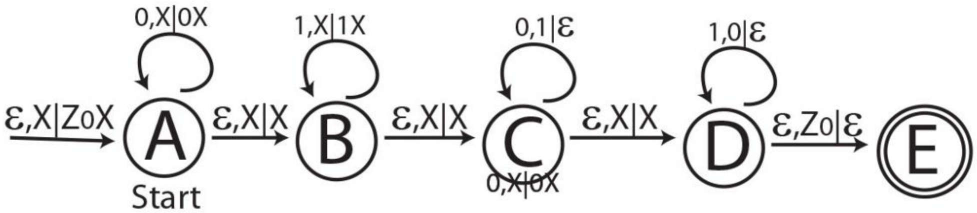
CS 383
Exam 2 Solutions

The exam has 6 questions. #1 is worth 20 points; the other five are worth 16 points each

1. Which of the following languages are context-free? Read the descriptions carefully. Write "CF" next to the languages that are context-free, "N" next to the ones that are not. No proofs are necessary.
   a. $\{0^n1^n0^n \mid n >= 0\}$ Not CF; a pumping lemma argument shows this.
   b. $\{0^n1^n1^n \mid n >= 0\}$ CF; this is the same as $\{0^n1^{2n} \mid n>=0\}$
   c. $\{0^n1^m1^m0^n \mid m, n >= 0\}$ CF; this is $\{0^n1^k0^n \mid n>=0$ and k is even$\}$
   d. Strings of 0s and 1s with odd length that have 0 as the middle element, such as 1110101 or 000 or 101. CF. Here's a grammar: S=> 0S0|0S1|1S0|1S1|0
   e. {vw | v is a string of 0s and 1s with length 3 or more and w is the first 3 letters of v} For example, 01101011 is a string in this language. CF; it is even Regular.
   f. $\{w0^n \mid$ w is a string of 0s and 1s and n is the length of w$\}$ For example, 01110000 is a string in this language. CF. Here's a grammar: S=>0S1|1S1|ε
   g. {uvw | u,v,w are all strings of 0s and 1s with the same length } For example, 011100 is a string in this language. CF. This is just the set of strings whose lengths are divisible by 3; it is Regular.

2. Construct a PDA that accepts by final state the language $\{0^n1^m0^m1^n \mid m, n >= 0\}$



3. Here is a grammar. Use this grammar to construct either a parse tree or a derivation (your choice; one is about as easy or hard as the other) for the string 001122:

   A ➔ 0A2| BC

   B ➔ 0B2 | C

   C ➔ 1A1 | 1

Here's a derivation:

A ➔ 0 A 2

➔ 0 0 A 2 2

➔ 0 0 B C 2 2

➔ 0 0 C C 2 2

➔ 0 0 1 1 2 2

4. Convert the following grammar to Chomsky Normal Form:

A ➔ 0A2| BC

B ➔ 0B2 | C

C ➔ 1A1 | 1

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B ➔ C is a unit rule; remove it

A ➔ 0A2| BC

B ➔ 0B2 | 1A1 | 1

C ➔ 1A1 | 1

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get rid of the terminal symbols in rules that have a mix of terminals and non-terminals

A ➔ ZAT| BC

B ➔ ZBT | NAN | 1

C ➔ NAN | 1

Z ➔ 0

N ➔ 1

T ➔ 2

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break down to rules of length 2

A ➔ ZA_1 | BC

A_1 ➔ AT

B ➔ ZB_1 | NA_2 | 1

B_1 ➔ BT

A_2 ➔ AN

C ➔ NA_2 | 1

Z ➔ 0

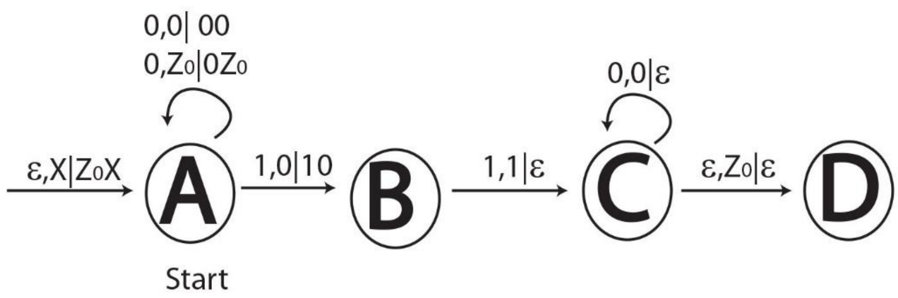
N ➔ 1

T ➔ 2

5. Give a careful pumping lemma proof that $\{0^i1^j2^k \mid 0<j,k<i\}$ is not a context free language. If you aren't clear about the language, it is the subset of 0*1*2* where there are more 0s than either 1s or 2s.

   Suppose this language is context free; let p be its pumping constant. Consider the string $z = 0^{p+1}1^p2^p$, which is in the language and longer than p. It should be pumpable. Consider any decomposition z=uvwxy where $|vwx| < p$ and vx is not empty. Since $|vwx| < p$ we know that v and x can't include all three digits. Suppose v and x together include at least one 0; then they include no 2. So uv^0wx^0y has no more 0s than 2s, so it isn't in the language. On the other hand, if v and x include no 0s then uv^1wx^1y has at least as many 1s or 2s as 0s, so it isn't in the language. In other words, for any decomposition of z there is some i for which uv^iwx^iy is not in the language, so z isn't pumable. This violates the pumping lemma, so our assumption that the language is context fee must be false.

6. In class we developed an algorithm by Noam Chomsky that constructs a grammar equivalent to a given PDA. Apply this algorithm to the following PDA and give the derivation in this grammar of the string 001100. Note that the PDA accepts by empty stack.



S $\rightarrow$ [A Z_0 D]

$\rightarrow$ 0 [A 0 C] [C Z_0 D]

$\rightarrow$ 0 0[A 0 C] [C 0 C] [C Z_0 D]

$\rightarrow$ 0 0 1[B 1 C] [C 0 C} [C 0 C] [C Z_0 D]

$\rightarrow$ 0 0 1 1[C 0 C} [C 0 C] [C Z_0 D]

$\rightarrow$ 0 0 1 1 0 [C 0 C] [C Z_0 D]

$\rightarrow$ 0 0 1 1 0 0 [C Z_0 D]

$\rightarrow$ 0 0 1 1 0 0

You can use this page as extra space for any problem.

Please write and sign the Honor Pledge when you have finished the exam. If you didn't take the exam with the rest of the class also write your starting and stopping times.